Last Example from Lecture 4 notes

Recall the structure of the data:

We chose the following model for the data:

$$y_{ij} = \eta_{1j} + \eta_{2j} x_{ij} + \beta_3 x_{ij}^2 + \varepsilon_{ij}$$
$$\eta_{1j} = \gamma_{11} + \gamma_{12} Girl_j + \varsigma_{1j}$$
$$\eta_{2j} = \gamma_{21} + \gamma_{22} Girl_j + \varsigma_{2j}$$

$$y_{ij} = \gamma_{11} + \gamma_{12} Girl_j + \varsigma_{1j} + \gamma_{21} x_{ij} + \gamma_{22} Girl_j x_{ij} + \varsigma_{2j} x_{ij} + \beta_3 x_{ij}^2 + \varepsilon_{ij}$$
$$y_{ij} = \gamma_{11} + \gamma_{21} x_{ij} + \beta_3 x_{ij}^2 + \gamma_{12} Girl_j + \gamma_{22} Girl_j x_{ij} + \varsigma_{1j} + \varsigma_{2j} x_{ij} + \varepsilon_{ij}$$
$$y_{ij} = \beta_0 + \beta_1 x_{ij} + \beta_2 x_{ij}^2 + \beta_3 Girl_j + \beta_4 Girl_j x_{ij} + \varsigma_{1j} + \varsigma_{2j} x_{ij} + \varepsilon_{ij}$$

In the last line above, we partitioned the fixed effects from the random effects.

Here is the table of output from fitting a random intercept and slope model were we first ignore gender and then when we take gender into account. Here assume age 0 is age at birth.

	Random Intercept and Slope		Random Intercept and Slope	
	Est	SE	Est	SE
_cons	3.49	0.14	3.75	0.17
Age	7.70	0.24	7.81	0.25
Age^2	-1.66	0.09	-1.66	0.09
Girl			-0.54	0.21
Girl*Age			-0.23	0.17
Random				
Tau_11	0.64	0.13	0.59	0.13
Tau_22	0.50	09.09	0.50	0.09
Rho_21	0.27	0.33	0.19	0.34
Sigma	0.58	0.05	0.57	0.05

I will interpret the output from the model that takes gender into account:

<u>Intercept (_cons):</u> The mean weight at birth for males is 3.75 kg (95% CI: 3.41 – 4.09).
<u>Age:</u> This is hard to interpret in the presence of the quadratic age term, but essentially the linear contribution to the age function for the males!
<u>Age^2:</u> Even harder to interpret, in general we don't interpret this. We do note that in this model, we assume that the quadratic component in the age function is the same for both genders.
<u>Girl:</u> This is the estimated difference in the weight of girls at birth relative to males at birth, they differ by 0.54 kg at birth with girls weighing less. The average weight for girls at birth is 3.75 – 0.54 = 3.21.
<u>Girl*Age:</u> This is the change in the linear component of the age function if the child is a girl. We will reduce the linear component of the age function by 0.23 kg per year for the girls.

<u>Tau_11 (random intercept):</u> Natural heterogeneity in weights at birth across children. We are assuming that the distribution of weights at birth are normally distributed with mean 3.75 kg for males and 3.21 kg for females. We expect that 95% of all birth weights will fall within +/- 2*0.59 = 1.08 kg of the mean for males and females, respectively.
<u>Tau_22 (random slopes):</u> Natural heterogeneity in how weights change with age across children. Technically, this is who the linear component of the age function varies across children. Again, we assume that childrens' linear slopes vary according to a normal distribution, we expect that 95% of all linear slopes to fall within +/- 2*0.50 = 1.00 kg/year of the true mean slope.

Rho_12 (correlation between random intercepts and slopes): The correlation coefficient is 0.19, indicating only a small correlation between a child's birth-weight and their linear slope.

NOTE: Correction here! We are inducing a heteroskedastic total variance for Y_ij
In this example Var(Y_ij) = tau_11^2 + tau_22^2 X_ij^2 + 2*tau_12 X_ij + Sigma^2.
But within subject variability is still only Sigma^2

Sigma:
Heterogeneity in weights for a subject at any given time.